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BRIEF COMMUNICATION

AN APPROXIMATE EXPRESSION FOR THE SHEAR LIFT FORCE ON A SPHERICAL PARTICLE AT FINITE REYNOLDS NUMBER

R. MEI

Department of Aerospace Engineering, Mechanics & Engineering Science, University of Florida, Gainesville, FL 32611, U.S.A.

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An expression for the shear lift force on a sphere was given by Saffman (1965, 1968) as

$$F_L = 6.46\rho\nu^{1/2}a^2(U - V)\left|\frac{dU}{dy}\right|^{1/2}\operatorname{sign}\left(\frac{dU}{dy}\right), \qquad [1]$$

where ρ and ν are the fluid density and kinematic viscosity, a is the radius of the sphere, U and V are the velocities of the fluid and the particle in the x-direction and dU/dy is the shear rate of the mean flow. In the derivation, it was assumed that

$$\mathrm{Re}_s = \frac{v_S 2a}{\nu} \ll 1, \qquad (v_S = |U - V|) \qquad [2]$$

$$\mathrm{Re}_G = \frac{G(2a)^2}{\nu} \ll 1, \quad \left(G = \left|\frac{dU}{dy}\right|\right) \qquad [3]$$

$$\mathrm{Re}_\Omega = \frac{\Omega(2a)^2}{\nu} \ll 1 \qquad [4]$$

and

$$\mathrm{Re}_S \ll \mathrm{Re}_G^{1/2} \quad \text{or } \epsilon = \frac{\mathrm{Re}_G^{1/2}}{\mathrm{Re}_S} \gg 1, \qquad [5]$$

where Ω is the rotational speed of the sphere. Equation [1] can be used with confidence only when the above conditions are met. However, practical situations arise during the study of particulate motion in turbulent flow which require an expression for the shear lift force at larger particle Reynolds number, Re_S, when conditions [2] and [5] are no longer met.

Recently, Dandy & Dwyer (1990) reported results for the shear lift force at a finite Re_S ($0.1 \leqslant \mathrm{Re}_S \leqslant 100$) and finite shear rate,

$$\alpha = \frac{Ga}{v_S} = \tfrac{1}{2}\mathrm{Re}_S\epsilon^2 \qquad [6]$$

($0.005 \leqslant \alpha \leqslant 0.4$). At $\mathrm{Re}_S = 0.1$, Saffman's prediction for the shear lift force, which was supposed to be asymptotically valid for large ϵ at small Re_S, was verified for ϵ as small as 0.447 (which corresponds to $\alpha = 0.01$). At $\alpha = 0.005$ ($\epsilon = 0.224$), Saffman's result is slightly larger than the numerical result [figure 3 of Dandy & Dwyer (1990)]. According to Dandy (1991), $\alpha = 0.005$ was the lowest value which was computed with reasonable numerical accuracy. Thus, for very small ϵ at low Re_S, it is not clear from the numerical result how Saffman's prediction differs from the actual value. As Re_S increases, it was found that the lift coefficient,

$$C_L = \frac{F_L}{\tfrac{1}{2}\rho\pi a^2 v_S^2}, \qquad [7]$$

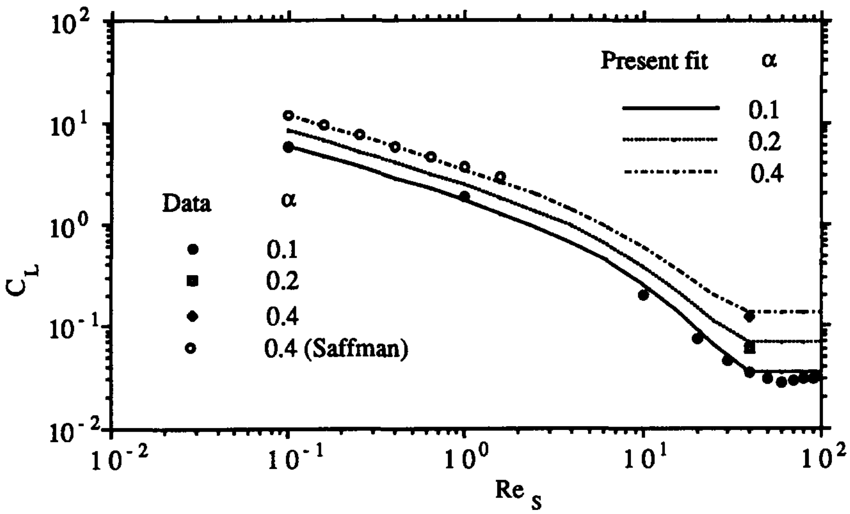

Figure 1 Comparison between the proposed expression, the numerical result of Dandy & Dwyer (1990) and Saffman's (1965) result for the shear lift coefficient

decreases. It levels off around $Re_S = 40$. At $Re_S = 40$, C_L increases linearly with α instead of with $\alpha^{1/2}$ as predicted by Saffman for small Re_S. The drag coefficient in the x-direction remains relatively insensitive to α.

After careful examination of the numerical results reported in Dandy & Dwyer (1990), the following approximation for the shear lift force at finite Re_S is proposed:

$$\frac{C_L}{C_{LSa}} = \frac{F_L}{F_{LSa}} = (1 - 0.3314\alpha^{1/2})\exp\left(-\frac{Re_S}{10}\right) + 0.3314\alpha^{1/2}, \quad Re_S \leqslant 40,$$
$$= 0.0524(\alpha Re_S)^{1/2}, \quad Re_S > 40; \qquad [8]$$

which combines the analytical result of Saffman (1965) at small Re_S and α and the numerical result of Dandy & Dwyer (1990). In the above, the subscript Sa denotes the corresponding result obtained by Saffman (1968). Figure 1 compares the shear lift coefficient, C_L, given by [8] with the asymptotic results of Saffman for small Re_S and finite α, and the numerical results of Dandy & Dwyer (1990). Satisfactory agreement can be observed.

It is interesting to compare a recent analytical finding on the shear lift force by McLaughlin (1991), for $Re_S \ll 1$ but arbitrary ϵ, with the numerical results of Dandy & Dwyer (1990) for $0.1 \leqslant Re_S \leqslant 100$. McLaughlin extended Saffman's analysis to include the effect of v_S in the outer region of the flow field. The effect of the sphere was replaced by a point force. The linearized momentum equation was solved in the wave number space and the lift force was evaluated numerically. The lift force for arbitrary ϵ can be expressed as

$$\frac{C_L}{C_{LSa}} = 0.443J(\epsilon). \qquad [9]$$

Saffman's (1965) result was recovered as $J \to 2.255$ for large ϵ. McLaughlin found that $J(\epsilon)$ decreases to zero rapidly as ϵ decreases, which means that Saffman's expression [1], would over-predict the actual shear lift force. From the table given in McLaughlin (1991), a curve fit for $J(\epsilon)$ is first constructed for $0.1 \leqslant \epsilon \leqslant 20$,

$$J(\epsilon) \approx 0.6765\{1 + \tanh[2.5 \log_{10}\epsilon + 0.191)]\}\{0.667 + \tanh[6(\epsilon - 0.32)]\}. \qquad [10]$$

It agrees very well with the result given in McLaughlin (1991). Next, it is used to compare the results of Saffman (1965), Dandy & Dwyer (1990) (represented by [8]) and of McLaughlin (1991) (represented by [9, 10]).

Using Re_S (rather than ϵ) as the abscissa and α as a parameter, the lift coefficients derived from [1], [8] and [9, 10] are shown in figure 2 for $\alpha = 0.1$ and 0.4. It can be seen that at low Re_S all three forms agree well with each other for $\alpha = 0.4$. At low shear rate, $\alpha = 0.1$, McLaughlin's result deviates quickly from the numerical result of Dandy & Dwyer as Re_S increases. This is expected because the asymptotic result of McLaughlin is valid only at low Re_S, while a decrease in ϵ means

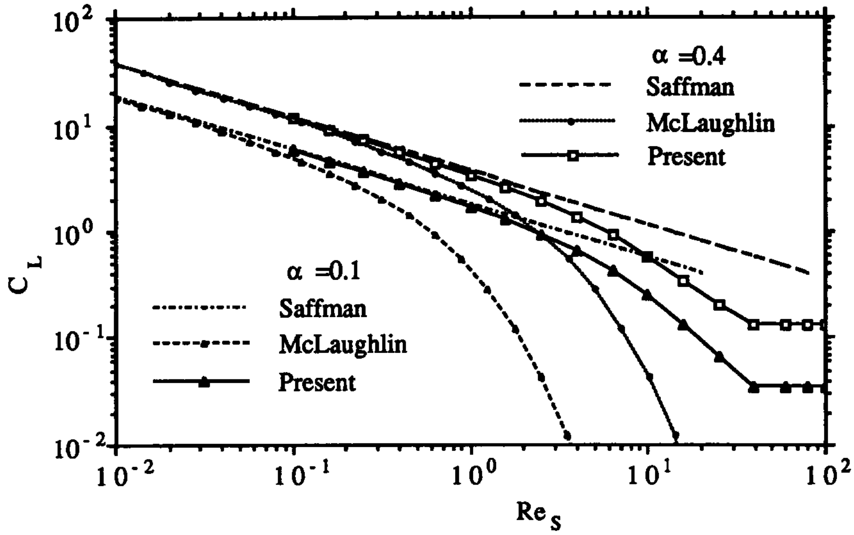

Figure 2. Shear lift coefficient based on the present approximate expression and the analyses of Saffman (1965) and McLaughlin (1991)

an increase in Re_S for a fixed α. Thus, one cannot expect the result to be accurate for a fixed shear rate α with decreasing ϵ. On the other hand, for a fixed Re_S (say $Re_S = 0.1$), McLaughlin's analysis indicates that the lift force decreases rapidly as α or ϵ decreases and deviates from Saffman's prediction, while the numerical result of Dandy & Dwyer at $Re_S = 0.1$ differs only slightly from Saffman's prediction even at $\alpha = 0.005$ and 0.01. It is not clear whether the discrepancy at $Re_S = 0.1$ and $\alpha \ll 1$ is due to the nonlinear inertia effect neglected in the analysis or to the numerical uncertainties, such as the size of the computational domain and grid resolution, in dealing with three distinctive regions of the flow field defined by Re_S and α.

Saffman's prediction for $\alpha = 0.1$ compares reasonably well with the numerical result up to $Re_S \sim 1$. This good agreement at $Re_S \sim O(1)$ can only be attributed to coincidence because there is no obvious reason, from a theoretical point of view, to expect [1] to perform better than [9, 10] in light of the analysis given in McLaughlin (1991). At higher Re_S, it is clear that both analyses fail to predict the correct C_L in comparison with the numerical result.

It should be mentioned also that the analysis of Saffman and that of McLaughlin are based on the assumption that $Re_\Omega \ll 1$. In the low Re regime, the rotation has little effect on the shear lift. The numerical result of Dandy & Dwyer is for $Re_\Omega = 0$. Thus, [8] can only be used, strictly speaking, for $Re_\Omega = 0$. The effect of rotation on the shear lift force at finite Re_S is not clear at present. In practical situations, particles do rotate in shear flow. Hence, [8] may be used as a guidance for small Re_Ω.

Since, to the best of the author's knowledge, there is no expression available as yet for the shear lift force on spheres, which can be used for the particle Re_S up to 100, it is hoped that the proposed expression [8], will be useful in the study of particle motion in the presence of significant shear at moderate particle Re_S.

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